A Graph of Classes Preserving Quantitative Temporal Constraints considering unbounded transitions

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Abstract

The objective of this paper is to extend the tool GraphC that generates a new graph of classes for t-time Petri nets, taking into account unbounded transitions. In this graph, a sequence of transitions effectively firable in the net is associated with each path between two nodes (classes). The constraints which have to be verified by the occurrence dates for any event sequence in the real system are directly derived by concatenating the constraints associated with the arcs covered by the corresponding sequence in the graph of classes.

1. Introduction

For checking some properties of critical embedded systems such as the timeliness property for correct environment interaction, it is frequently necessary to consider specific scenarios of operations and to analyze the temporal constraints which have to be verified by the events composing them [Ri 01].

Other properties (related for example to the fact that a state is not reachable) imply the exhaustive search for all the states of a system. When temporal constraints exist, the states, in an infinite number, can be covered by a finite set of state classes for bounded Petri nets. In this case, a graph of state classes can be built in order to study the system, where nodes are state classes and the arc from a class $C$ to a class $C'$ is labeled by the transition $t$ (leading from $C$ to $C'$).

Several kinds of classes have been proposed according to the kind of properties to be proven (properties expressed in LTL or in CTL for instance) [Yo 98, Be 04, Ca 05].

In order to correctly delimit the domains of the variables attached to the firing dates in a transition firing sequence, it is necessary, in the case of a t-time Petri net [Be 04] with strong semantics, to know the transition enabling dates. This implies that, for each transition, the date of the firing which has produced the last token is known. In consequence, it is necessary to proceed in the context of interleaving semantics and therefore to explicitly consider states and firing sequences.

In this paper, the proposed approach is to construct a graph of classes with sets of constraints attached to its arcs, such that the constraints which have to be verified by the firing dates for any sequence in the net, are directly derived by concatenating the constraints attached to the arcs covered by the corresponding sequence in the graph of classes.

2. Basic notions

2.1. Simple temporal network (STN)

A STN $N$ is composed of a finite set $V$ of variables $v_i$ and a finite set $C$ of binary constraints $C_i j(v_i,v_j)$ defined as convex intervals $[c_{mi j}, c_{Mij}]$ delimiting the possible distance between two variables $v_i$ and $v_j$ of $V$.

A STN $N = (V,C)$ is complete iff a constraint $C_i j$ is associated with each pair of variables. A complete STN is minimal iff $\forall v_i, v_j \in V$ and $\forall c \in C_{i j}, c \in [c_{mi j}, c_{Mij}]$, is such that $v_j - v_i = c$. The Floyd-Warshall algorithm derives from any consistent (having at least one solution) STN [De 91] a new complete and minimal STN.

2.2. t-time Petri nets

Definition 1 A t-time Petri Net is a 3-tuple $< N, M_0, I >$:

- $N = < P, T, Pre, Post >$ is a Petri net,
- $M_0$ is the initial marking,
- $I : T \rightarrow (Q^+ \cup 0) \ast (Q^+ \cup \infty)$.

The static interval function $I$ associates with each transition $t_i$ a temporal interval $[a_i, b_i]$ (see fig. 3.a) that represents the set of its possible firing dates. When the upper bound of $I(t)$ is $\infty$ it is said to be unbounded. Otherwise it is bounded.

In this paper, the operational semantics for t-time Petri nets, includes the strong semantics (which enforces the firing of one of the enabled transitions before the earliest of all the latest firing dates for the enabled transitions) and the interleaving semantics (transitions may be enabled concurrently but are fired sequentially). It is assumed that there is no memory of the enabling time of a transition in the past.
In a t-time Petri Net, the following events associated with a transition must be taken into account: the enabling date, begin/end of the firing interval and firing date. The following constraints must be verified between these events:

- the enabling date of a transition \( t \) is equal to the firing date of the last transition \( t' \) contributing to its enabling,
- the transition firing date should be included in its firing interval \( I \).

### 3. The graph of classes

#### 3.1. States and state classes

Let us consider the execution of a firing sequence \( \sigma = t_1 \cdots t_i \cdots t_n \) in a t-time Petri net with unbounded transitions. A transition can be fired several times in a sequence. The \( j \)-th firing in \( \sigma \) of transition \( t_i \) is denoted by \( x_i^j \) (if \( o_i = 1 \), we note \( x_i \) instead of \( x_i^j \)).

Given a specific execution of \( \sigma \), the state after the firing of \( t_i \) is obtained marking associations with the current value of the clock and the firing dates of all the transitions preceding \( t_i \) in \( \sigma \) in order to compute the remaining firing intervals for each enabled transition.

A class is composed of all the states which are reachable by an execution of \( \sigma \) after the firing of \( t_i \) and before that of \( t_j \). Aiming to have all the constraints which must be verified by the firing date of \( t_i \), it is necessary to be able to derive not only the distance of \( x_i^j \) and \( x_j^j \), but also the distance of \( x_i^j \) with all the preceding firing dates in \( \sigma \). In order to have a finite number of classes, it is necessary to forget a part of the past, keeping only a fragment of the STN made by these variables and their constraints.

The initial state class \( C_0 = (M_0, N_{C_0}, T_{0c}^\infty) \) is given by: the initial marking \( M_0 \), the STN \( N_{C_0} : x_0 \), where \( x_0 \) represents the time origin (the beginning of the world) and the set of unbounded transitions \( T_{0c}^\infty = \{ t \mid I(t) = [0, \infty) \} \).

Let \( \sigma \) be a firing sequence \( t_1 \cdots t_i \cdots t_n \) of a t-time Petri net, \( t_i \) the last fired transition in \( \sigma \) and \( t_{s(k)} \) the transition that has enabled a transition \( t_k \).

**Definition 2** The state class \( C \), obtained after the firing of transition \( t_i \), is defined by \( M, N, T_{\infty}^c \) where:

- \( M \) is the current marking of the net; it is assumed that \( n \) transitions are enabled by \( M \),
- \( T_{\infty}^c \) is the set of unbounded transitions not constrained by the firing of \( t_i \),
- \( Nc \) is the minimal and complete STN composed of the following variables and constraints:
  1. the variable \( x_i^j \) associated with the last transition firing \( t_i \) (firing),
  2. for each enabled transition \( t_k \) from \( M, t_k \notin T_{\infty}^c \), the variable \( x_{s(k)} \) \( (k = 1, \ldots, n) \) associated with the firing of transition \( t_{s(k)} \).

3. the temporal constraints between these variables (minimal and complete network).

The complete definition of the temporal network \( Nc \) requires the initial constraints in point 3. These values are taken from the STN defined in the section 3.2.

When \( t_k \) is enabled by a transition \( t_{s(k)} \) at some class before \( C \) (def. 2), its initial enabling time is the static interval \( I(t_k) \). If \( t_k \) remains enabled at \( C \) (after \( t_i \) firing), the possible firing dates of \( t_k \) are no longer delimited by possible firing dates of \( t_k \) but by the dynamic interval \( I_d(t_k) \). In order to take the same time origin \( x_{s(k)} \) than \( I(t_k) \), \( D(t_i) = C_{s(k),i} \) and (time has non negative values):

\[
I_d(t_k) = (I(t_k) - C_{s(k),i}(x_{s(k)}, x_i)) \cap [0, \infty) \tag{1}
\]

#### 3.2. The arc associated with the firing of \( t_j \)

In our approach, the arc \( (C, C') \), leading the system from class \( C \) to \( C' \) with the firing of transition \( t_j \), is labelled by \( t_j \) and is associated with:

- a set of unbounded transitions not constrained by the firing of transitions leading to \( C \), \( T_{\infty}^c = T_{\infty}^c \mathbin{\setminus} \{ t_j \} \),
- a STN \( Nt_{j,C} \) delimiting the firing date of \( t_j \) from \( C \). It reflects the memory of the past necessary to characterize the dates of the future events.

Let \( t_j \) be a transition among the \( n \) enabled transitions at class \( C = \{ M, Nc, T_{\infty}^c \} \), with \( Nc = (Vc, Cc) \), and let \( t_l, l \neq j \), be the other \( n-1 \) enabled transitions at \( C \).

**Definition 3** The STN \( Nt_{j,C} = (Vt, Ct) \) delimiting the firing of \( t_j \) from class \( C \) is composed of:

1. all \( Nc \) variables and constraints, \( Vt = Vc, Ct = Cc \);
2. the variable \( x_j^j \) (firing date of \( t_j \)) and the static interval \( I(t_j) \) as a constraint between \( x_{s(j)}^j \) and \( x_j^j \),
3. the variable \( y_j^j \) corresponding to each bounded transition \( t_j \), with the constraint \( C_{s(j),i}(x_{s(j)}^j, y_j^j) = [d_{Ml}, d_{Ml}], \) the upper bound of static interval \( I(t_i) \),
4. the variable \( z_j^j, l \neq j, l \notin T_{\infty}^c, \) corresponding to each enabled unbounded transition \( t_j \) with the constraint \( C_{s(j),i}(z_j^j, y_j^j) = [d_{Ml}, d_{Ml}], \) the lower bound of static interval \( I(t_i) \) \( t_i \) \( \{\{t_i\}\} + \{\{y_j^j\}\} = n - 1 \).
   If \( c_{Mj}(x_j^j, y_j^j) \geq 0 \), \( T_{\infty}^c = T_{\infty}^c \cup \{ t_j \} \).
5. \( C_{j,i}(x_j^j, x_j^j) = [0, \infty) \) to express the fact that \( t_j \) must be fired after \( t_i \) (interleaving semantics),
6. \( C_{j,i}(x_j^j, y_j^j) = [0, \infty), l \neq j, \) to express the fact that \( t_j \) must be fired before the upper bound of the firing interval of bounded transitions \( t_i \) (strong semantics).
If after applying Floyd-Warshall algorithm, \( Nt_{j,c} \) is consistent, \( t_j \) can be fired. The final \( Nt_{j,c} \) is obtained deleting all nodes \( y_i \) and \( x_{ij} \) (if all the other transitions enabled by \( x_{ij} \) are in \( T_{\infty}^c \)). All constraints directly connected to the deleted variables are also deleted [Ma 05].

Let us consider the Petri net of fig. 1.a, with initial class \( C_0 = \{ p_1 p_2, x_0, \emptyset \} \). The firing of \( t_j \) from \( C_0 \) leads to class \( C_1 \); this arc has \( Nt_{1,0} = \emptyset \). The final \( Nt_{1,0} \) is given by \( V = \{ x_0, x_1 \} \) with \( C_{0,1} = \{ [1,1] \} \). Class \( C_1 \) has \( M_1 = p_1 p_2 \), \( T_{\infty}^c = \emptyset \) and \( Nc_1 \) given by \( V = \{ x_0, x_1 \} \) with \( C_{0,1} = \{ [1,1] \} \).

The firing of \( t_2 \) from \( C_0 \) leads to class \( C_2 \) and is associated with \( Nt_{2,0} \) (fig. 1c). The final \( Nt_{2,0} \) is only given by node \( x_2 \) and \( T_{\infty}^c = \emptyset \) and \( T_{\infty}^c \in \{ \{1\} \} \). Class \( C_2 \) with \( M_2 = p_1 p_2 \), set \( T_{\infty}^c = \{ t_1 \} \) has \( Nc_2 \) given only by node \( x_2 \).

3.3. Restricted class

Some constraint \( C_{k,l} \) between two nodes \( x_k \) and \( x_l \) in the \( Nt_{j,c} \) from the class \( C \), can become more restricted than in the network \( Nc \) of \( C \). This means that transition \( t_j \) can only be fired from the states of \( C \) for which variables \( x_k \) and \( x_l \) verify this new, more restricted constraint \( C_{k,l} \). This defines a sub-class \( C_{r,j} \) restricted in order to permit the firing of \( t_j \).

Let \( t_j \) be a transition which can be fired from \( C = (M, Nc) \) whose firing date is delimited by the \( Nt_{j,c} \).

Definition 4 The restricted class \( C_{r,j} = (M_r, Nc_r) \) of class \( C \) is created if \( Nt_{j,c} \cap Nc_r \neq Nc_r \) and is defined by:

- i) \( M_r = M \)
- ii) \( T_{\infty}^c = T_{\infty}^{c_r} \)
- iii) \( Nc_r = Nt_{j,c} \cap Nc \)

After a new application of Floyd-Warshall new restricted classes can appear in the past.

3.4. Equivalent classes

Definition 5 Two classes \( C = (M, Nc) \) and \( C' = (M', Nc') \), differing from the initial class \( C_0 \), with \( Nc = (X, C) \) and \( Nc' = (X', C') \) are equivalent if:

1. they have the same marking, \( M = M' \) and the same set \( T_{\infty} = T_{\infty}' \)
2. there exists a bijection \( \tau \) between the elements of \( X \) and \( X' \) such that:
   - i) \( x_k^{(r)} \) \( \tau(x_k^{(i)}) \) implies \( k = i \) (they are firing dates of the same transition);
   - ii) if \( x_k = \tau(x_i) \) and \( x_j = \tau(x_j) \) then \( C_{ij}(x'_i, x'_j) = C_{ij}(x_i, x_j) \).

Definition 6 A class \( C = (M, Nc) \) is equivalent to the initial class \( C_0 = (M_0, x_0) \) if: i) \( M = M_0 \) and \( T_{\infty} = T_{\infty}^c \), 2) the set of variables \( X \) of \( Nc \) is a singleton, \( X = \{ x_k \} \) (no past memory).

In fact, the firing of \( t_k \) leads the system back to the initial marking \( M_0 \) and enables all transitions at this marking.

The firing of \( t_2 \) in fig. 1 from \( C_2 \) leads to a class \( C' \):

\[ T_{\infty}^{c_r} = \{ t_1 \} \]

\[ Nt_{2,2} \] represented in fig. 1d. Class \( C' \) has \( M' = M_2 \) and \( Nc' \) given by \( x_2 \), so it is equivalent to \( C_2 \) (def. 5). The classes and the graph are represented in fig. 2 (\( C_7 \) is a restricted class of \( C_4 \)). All classes have \( M = p_1 p_2 \).

3.5. Sequence characterization

The temporal network of a sequence \( \sigma = t_1; \ldots; t_i; t_j; \ldots; t_n \) from a class \( C \) is given by \( Nt_{\sigma} = Nt_{1,c} \cup \ldots \cup Nt_{j,c} \cup \ldots \cup Nt_{n,c,n} \) [Ma 05].

A particular case appears when \( \sigma = t_1; \ldots; t_i \) and the firing of last transition \( t_i \) leads to a class \( C \) and also to its restricted class \( C_r \). Two networks are obtained, \( Nc_{r1} \) leading to \( C \) and \( Nc_{r2} \) leading to \( C_r \), but \( Nc_{r2} \subseteq Nc_{r1} \) and so the network characterizing \( \sigma \) is \( Nc_{r1} \) (see an example in the sequel).

4. Example

Let us consider an unidirectional protocol of data transfer [Ca 05] modeled by the Petri net in fig. 3.a presenting infinite sequences. We want to know if an overwrite, due
to the earlier arriving of a new message whereas the precedent was not yet consumed, can occur in this system (represented by transition \( t_4 \)). The underlying Petri net (the structure without time specifications) is unbounded and so it is not possible to know if the overwrite is done. The graph is represented in fig. 3.b and the classes are given below:

<table>
<thead>
<tr>
<th>Class</th>
<th>Marking</th>
<th>Constraints ( N_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( P_1 \times P_6 \times P_4 )</td>
<td>( C_1 ) = ([0,3])</td>
</tr>
<tr>
<td>1</td>
<td>( P_1 \times P_5 \times P_6 )</td>
<td>( C_2 ) = ([1,5] )</td>
</tr>
<tr>
<td>2</td>
<td>( P_1 \times P_5 \times P_6 )</td>
<td>( C_3 ) = ([0,3])</td>
</tr>
<tr>
<td>3</td>
<td>( P_1 \times P_5 \times P_6 )</td>
<td>( C_4 ) = ([2,4])</td>
</tr>
<tr>
<td>4</td>
<td>( P_1 \times P_5 \times P_6 )</td>
<td>( C_5 ) = ([0,2])</td>
</tr>
<tr>
<td>5</td>
<td>( P_1 \times P_5 \times P_6 )</td>
<td>( C_6 ) = ([2,3])</td>
</tr>
</tbody>
</table>

We present here only some temporal network \( N_t \) attached to the arcs:

\[
\begin{align*}
N_{t_0} &= \{(x_0, x_1) = [4, 6], N_{t_1} = \{(x_1, x_2) = [2, 3]\} \}, \\
N_{t_2} &= \{(x_1, x_2) = [2, 3], (x_2, x_3) = [0, 6]\} \}, \\
N_{t_3} &= \{(x_2, x_3) = [2, 3], (x_3, x_4) = [4, 5], (x_4, x_5) = [1, 2]\} \}.
\]

Which constraints must be met in order to fire \( t_4 \) (overwriting)? Let us consider a sequence including the firing of \( t_4 \) in the graph of fig. 3.b, for example, \( \sigma = t_1 \cup t_2 \cup t_3 \cup t_1 \cup t_2 \cup t_4 \). The network delimiting \( \sigma \) is given by

\[
N_\sigma = N_{t_1} \cup N_{t_2} \cup N_{t_3} \cup N_{t_2} \cup N_{t_1} \cup N_{t_4} \cup N_{t_6}.
\]

This network allows obtaining the exact temporal constraints that have to be verified by each transition firing with respect to a given firing sequence. It extended the graph presented in [Ma 05] considering unbounded transitions. A state class reached by the firing of \( t \) in the graph is defined by a marking, a temporal network and a set of transitions not constrained by the firing of \( t \). An arc between two classes is labeled by a temporal network \( N_t \) delimiting the firing date of a \( t_i \) itself and a set of transitions not constrained by the firing of transitions leading to the source class. The temporal constraints verified by a firing sequence are obtained by the union of the temporal constraints \( N_t \) attached to each arc along the corresponding path on the class graph.

There are two main differences with the graphs proposed in [Yo 98, Be 04]. The first one is that we use simple temporal networks instead of geometrical regions to deal with temporal information. The second one appears in the way the past is memorized. In relation to [Yo 98], our class does not keep all the constraints in the past, but only the ones that are necessary to characterize it, as proven in [Ma 05], and consider unbounded transitions. In relation to [Be 04], we can directly obtain the set of constraints of a sequence instead of obtaining it by transformations and calculations.

Further research should consider to extend the graph of classes to deal with time fuzzy Petri nets, where the interval of firing is fuzzy, allowing to evaluate a possibility and necessity degree of transition firing.

5. Conclusion

The presented approach instead of graphs of classes that allows obtaining the exact temporal constraints that have to be verified by each transition firing with respect to a given firing sequence. It extended the graph presented in [Ma 05] considering unbounded transitions. A state class reached by the firing of \( t \) in the graph is defined by a marking, a temporal network and a set of transitions not constrained by the firing of \( t \). An arc between two classes is labeled by a temporal network \( N_t \) delimiting the firing date of a \( t_i \) itself and a set of transitions not constrained by the firing of transitions leading to the source class. The temporal constraints verified by a firing sequence are obtained by the union of the temporal constraints \( N_t \) attached to each arc along the corresponding path on the class graph.

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References


