Relationships between Petri nets and constraint graphs: application to manufacturing

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Objective

• (Time(d)) Petri nets seen as (T)Constraint Satisfaction Problems
• TCSP algorithms and Constraint Propagation used in the context of Petri nets
• Clear separation between discrete (logic) and continuous (time)
• Reverse the classical approach (Linear prog. and then Tree based search) : Discrete constraints are satisfied first
• Simultaneously solve Assignment and Scheduling
Ordinary Petri net and CSP

- Petri net defines precedence relations in a procedural way
  - They are not explicitly enumerated

- Two consistent sets of precedence relations
- Avoid interleaving (firing sequence)

Linear logic and Petri nets

- Equivalence between reachability in Petri nets and the provability of specific sequents in Linear logic (Girard's)
- Tokens have to be produced before being consumed
- Checked during the proof => precedence relations derived
- One proof => one consistent set of precedence relations (A fragment of one process in the Petri net unfolding)
- One proof = a sequence of decisions i.e. conflict (tokens and transitions) resolutions
An example

Conflict t2 t4 : decision
Two proof trees
Two partial orders

Introducing time

• If one consistent set of precedence relations has been selected
time constraints are introduced in a linear way (AOA-graphs)

• The following steps are executed:
  – Petri net model (static definition of manufacturing system and specification of how
    resource assignments can be done)
  – From the current marking to another marking => defines an assignment/scheduling
    problem
  – Derive one partial order (heuristic based?) => a set of consistent precedence relations
  – Take continuous time constraint into account => Activity on arcs graphs (AOA)
  – Use Linear Programming or Constraint Propagation (arc consistency) on AOA
p-timed Petri nets $\Rightarrow$ AOA

- $d_i$ minimal sojourn time in place $p_i$
- Transition firing nodes $\Rightarrow$ firing dates (variables)
- Arc labels (place names) $\Rightarrow$ constraints (duration: $x_3 \geq x_2 + d_i$)

p-time Petri net $\Rightarrow$ AOA

- $d_{im} \text{ minimal, } d_{IM} \text{ maximal sojourn time in place } p_i$
- Transition firing nodes $\Rightarrow$ firing dates (variables)
- Arc labels (places) $\Rightarrow$ constraints (duration) (2 arcs each place $x_3 \geq x_2 + d_{3m}$ and $x_3 \leq x_2 + d_{3M}$)
t-timed Petri nets $\Rightarrow$ AOA

- $d_i$ firing duration attached to each transition
- Each node broken down into 2: $x$: "begin firing" $y$: "end firing"
- Transition arc ($y_2 \geq x_2 + d_2$), token arc ($x_3 \geq y_2$)

![Diagram of t-timed Petri nets](image)
**Conclusion**

- **Comparing the time extensions of Petri nets / constraints:**
  - p-timed, t-timed and t-time Petri nets are such that derived AOA-graphs are without positive oriented circuits, "logical" reachability entails the existence of at least one sequence verifying all the temporal constraints.
  - t-time Petri nets are not adequate because it is necessary to know the firing sequence (combinatorial explosion).
  - p-time Petri nets are the more general (possibility of temporal inconsistencies) and AOA graphs have the same structure as the precedence graphs.

- **From a Petri net model: simultaneous assignment + scheduling**

- **A basis to elaborate hybrid approaches (Petri net + Constraints)**
p-timed Petri nets $\Rightarrow$ AOA

p-time Petri net $\Rightarrow$ AOA
t-timed Petri nets => AOA

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