2. The formal framework provided by Petri nets

"Batch processes and hybrid systems" aspect

The issue (1)

Time and mathematical tools:

- **Differential and algebraic equations**
  - continuous processes in their steady state are described by algebraic equations
  - for the dynamic behavior differential equations => DAEs
  - derive of a variable

- **Systems with discrete states**
  - a decision in one state: decision trees, boolean expressions, classical logic
  - dynamic system: finite state automata, Petri nets
  - next state = qualitative notion of time, precedence relation
The issue (2)

Types of math. models with time

- **Continuous systems**
  - continuous state variables - continuous time

- **Sampled systems**
  - continuous state variables - discrete time

- **Discrete systems**
  - discrete state variables - continuous time

- **Discrete event systems**
  - discrete state variables - discrete time

The issue (3)

Batch and hybrid systems:

- **Ordinary Petri nets**
  - discrete event model, discrete state variables, discrete time = qualitative = logic

- **Variables and time**
  - discrete variables for configurations (steps), continuous variables (batch size, quality)
  - discrete time for sequences of discrete operations, continuous time for continuous op.

How building a model for these systems?
Kind of Petri nets (1)

With respect to time:

- **Timed and time Petri nets**
  - explicit duration, qualitative, time takes value in integers or rational numbers
  - decidability - but in practice time is real number, stochastic Petri nets

- **In simple case variables are proportional to time**
  - constant flow rate: quantity of material transferred prop. with duration

**Time (timed) PN first abstraction for hybrid systems**


Kind of Petri nets (2)

With respect to variables:

- **Coloured and high-level Petri nets**
  - finite set of colours: decidability, unfolding into ordinary Petri nets
  - object PN: attributes on tokens as real numbers, attribute transformation as methods
  - in simulation tools (Design/CPN, MISS-RdP) pieces of code attached to trans. + time

=> **Continuous variables and time**

- discrete variables = places - discrete time = precedence relations (arcs)
- continuous var. = token attributes - cont. time = quantitative val. (trans. or places)

Kind of Petri nets (3)

Hybrid Petri nets:

- **Limitation of coloured Petri nets**
  - tokens attributes are only updated at the occurrence of transition firings
  - continuous variables are not continuously known (kind of sampled model)
  - management of data at different clocks => each Δt transitions are fired

  => **Data have to be continuously known or linear interpolations**
  - discrete variables = disc. places - discrete time = transition firings
  - continuous var. = cont. places - cont. time = linear interpolation between firings


Kind of Petri nets (4)

Petri nets with Differential Algebraic Equations (DAE):

- **Limitations of hybrid Petri nets**
  - derivates have to be constant on time intervals
  - thresholds are "Petri net like" (at least value "v")
  - no algebraic constraints on the continuous variables

- **Continuous part described by DAEs**
  - DAE are "controlled" by Petri net marking evolutions
  - in a marking, DAE solver computes cont. variables => thresholds => fire trans.

  >> R. Champagnat, P. Esteban, H. Pingaud, R. Valette: Modelling and simulation of a hybrid system through Pr/Tr PN and DAE, ADPM 98 p.131-137, Reims, March 1998