A gas storage example as a benchmark for hybrid modelling: a comparative study

Une étude comparative de modélisation hybride sur un exemple de stockage de gaz

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ABSTRACT. This paper presents a comparative study of various approaches for modelling hybrid systems by means of Petri nets. The comparison is based on a simplified view of an industrial device: a gas storage unit. This example illustrates the advantages and limits of the discrete-time based approach offered by Coloured Petri nets, those of Hybrid Petri nets when some continuous variables are connected by algebraic constraints and those of Petri nets with differential algebraic equations when some continuous variables are shared by various sub-models.

RESUME. Cet article présente une étude comparative de diverses approches de modélisation de systèmes hybrides au moyen des Réseaux de Petri. La comparaison est fondée sur une vue simplifiée d'un équipement industriel : une unité de stockage de gaz. Cet exemple montre les avantages et limites de l'approche à temps discret offerte par les réseaux de Petri Colorés, celle des réseaux de Petri Hybrides lorsque des contraintes algébriques reliant des variables d'état continues et celles des réseaux de Petri associés à des équations algébro-différentielles lorsque des variables continues apparaissent dans plusieurs sous-modèles.

KEY WORDS: Hybrid model, Petri nets, Hybrid Petri nets, Benchmark, Differential Algebraic Equations

MOTS-CLES : Modèles hybrides, Réseaux de Petri, Réseaux de Petri Hybrides, Etude comparative, Equations Différentielles Algébriques
1. Introduction

The objective of this paper is to compare various approaches for modelling hybrid systems in order to derive some useful guidelines which could be used by engineers to predict the behaviour of hybrid systems. It is the result of a collaboration between four research institutes (LAAS-CNRS, LAG-ENSIIEG, LAGEP-UCB, LGC-ENSIGC). The first aim of this collaboration is the definition of a set of typical examples which could be used to analyse the differences between various modelling approaches. Among these examples, a gas storage unit has been chosen because the continuous view is sufficiently complex to illustrate some hybrid issues, as it involves one differential equation and also non-linear algebraic ones. On the other hand, the discrete part is less complex than, for instance, in typical batch systems. This benchmark is inspired from a case study proposed by ELF Aquitaine within the framework of the ANVAR project "hybrid simulation" [LAN 97]. This ANVAR project which involves research institutes and industrial end users under the leadership of the consultant firm IXI, aims at developing a simulation tool for the performance evaluation of systems having discrete and continuous dynamics.

As various ways of addressing this issue could be considered, the four laboratories agree that the discrete part should be represented either by Petri nets or by a model of the same family such as the Grafet, and that the continuous part should be added or integrated in some way to it. When the continuous part is very simple, it can be taken into account by adding timing considerations (state events are transformed into time event by encapsulating the trajectory of continuous variables from some initial state to some thresholds by a duration). In this case, time or timed Petri nets are sufficient [SIF 77, MER 76, HAN 93].

When the continuous trajectories of the variables cannot be encapsulated into a simple constant duration, basically three approaches have been investigated:
- either the duration is dynamically computed in function of some token attributes in a coloured Petri nets [GEN 98],
- or the dynamics of the continuous part is captured within the same formalism as the discrete part (Hybrid Petri nets) with continuous places which are used to code the continuous variables by considering that their token load is a real number instead of an integer [DAV 87, FLA 96],
- or the dynamics of the continuous part is expressed by the traditional way \textit{i.e.} differential algebraic equations; the way these equations interact with a Petri net or a Grafet have to be clearly specified [AND 94, DAU 94, VAL 93, VIB 97].

The intuition is that the first approach cannot efficiently capture complex continuous dynamics, the second offers a clear representation of the continuous flow of material by a continuous token flow and the third does not require restriction about differential algebraic equations describing the dynamics. We have been motivated to define some benchmarks in order to better compare the benefits and drawbacks of each approach and it has been clear that an example with a rich continuous part was essential. The gas storage facility happened to present the required characteristics.

After a presentation of the gas storage example, three Petri net based approaches for modelling hybrid systems are compared in section 3. Then in section 4, some difficulties which are pointed out by the benchmark example are discussed.
2. Presentation of the example

This example comes directly from the ANVAR project. Its general structure is represented in figure 1. It is composed of a storage unit S (defined by pressure $P_s$, volume $V_s$ and number of moles $U_s$), a compressor $C_p$ and a compressor $C_c$. As the issue which was to be pointed out was the case of systems having a non trivial continuous part, its discrete part is a simplified version of the original one which includes a pool of compressors. This pool of compressors has been reduced to two: one at the interface with the production and one at the interface with the consumers. In addition the possible failures have been neglected. Naturally, the values of all the parameters are not the actual ones (because of industrial restrictions).

![Figure 1. Gas storage unit](image-url)

2.1. General description

The gas storage is located between a production unit and a customer unit. Its goal is to introduce a buffer in order to facilitate the balance between production and customer demand.

The storage is fed by the production unit at pressure $P_p$ and flow rate $d_p$; it is drawn by the customer unit at pressure $P_c$ and flow rate $d_c$. Temperature is supposed to be constant and equal to 298 K ($T_s = T_c = T_p = T = 298$ K).

The process is composed of three main functions: upstream compression, storage and downstream compression.

2.1.1. Upstream compression

The upstream compression is situated between the production unit and the storage tank. Its goal is to adapt pressure between the production and the storage unit. This function is used if the pressure $P_p$ is not high enough to ensure that the gas is flowing from production to storage (where pressure is $P_3$) and not in the opposite direction. If needed, $V_2$ is closed and $V_1$ is open. The compressor produces a constant specific work whatever the flow rate is. The output pressure is therefore equal to the input pressure multiplied by the compression rate. Valve $V_{R1}$ is required to connect the output of the compressor (pressure $P_2$) with the input of the storage unit (pressure $P_3$). The energy dissipation resulting from the flow rate $d_p$ through $V_{R1}$ compensates the pressure difference $P_2 - P_3$. 
2.1.2. Storage

The originality of this example results from two properties of the storage unit. The first is that it is a *hydraulic subterranean storage* (see figure 2) which means that the storage capacity depends on its internal pressure. The second property comes from the fact that the material stored is a *compressible gas*. This means that the molar capacity of the storage \((U_s)\) depends on pressure, temperature and volume. This storage cannot be simultaneously fed and drawn. But the gas can flow through \(V_7\) and the by-pass linking the production and the customer unit. Valves \(V_3\) and \(V_4\) are mutually exclusive. If the first is opened the second must be closed. When the flow rate is divided between the by-pass and the tank, either valve \(V_{R1}\) or \(V_{R2}\) is used to ensure a consistent distribution of the flow.

![Figure 2. Simplified view of the gas storage](image)

2.1.3. Downstream compressor

The downstream compression, which is identical to the upstream one, is used to ensure that \(P_5\) is greater than \(P_C\) and therefore to maintain the flow direction from the storage unit to the customer and to prevent any flowing back.

2.2 Configurations of the production unit

The gas storage cannot be fed and emptied simultaneously. This is due to safety considerations and design issues of the storage unit itself. As a consequence, the storage unit must only be driven according to one of the four following configurations.

2.2.1. Configuration 1
In the first configuration, the production flow rate and the customer flow rate are identical (figure 3). The storage is by-passed and input compressor and dissipator are off. Actually a similar result can be obtained by putting the output compressor and dissipator off and operating with the input ones.

![Figure 3. Configuration 1](image)

2.2.2. Configuration 2

The second configuration is active if the production flow is greater than the customer flow. The gas goes from the production unit to the customer unit and the surplus is filling the storage (figure 4).

![Figure 4. Configuration 2](image)

2.2.3. Configuration 3

In the third configuration, the production flow is lower than the customer flow. The produced gas flows to the consumers and the lack of gas is compensated by emptying the storage (figure 5).

![Figure 5. Configuration 3](image)

2.2.4. Configuration 4
The fourth configuration is active if the downstream gas flows from the storage only. The production unit is disconnected by means of valve V7, and input compressor and dissipator are off (figure 6).

2.3. Gas storage equations

As mentioned above, the example complexity comes from the compressible nature of the gas stored in a hydraulic subterranean storage. This means that the volume of the storage depends on its pressure. As the quantity of gas increases, the liquid surface in the storage is pushed down, following the hydrostatic law.

Figure 2 shows, in a simple way, the physical principle of the volume variation of the storage. The sum of the height of the two phases, \( h_g + h_l \), remains constant, and the gas volume is determined by the height \( h_g \). The pressure at the liquid/gas interface of the tank is determined by the hydrostatic equilibrium column. The following equation can be derived:

\[
P_s = P_0 + \rho_l \cdot g \cdot h_s
\]

where \( P_0 \) is a constant (reference pressure) and \( \rho_l \) is the volumic mass of the liquid. If the pressure \( P_s \) increases, the surplus height \( h_s \) increases too. This implies, by a simple material balance, that \( h_l \) decreases. Then the gas pressure increases and its volume also.

Since the interior shape of the storage is irregular, the volume evolution obeys to a non-linear law

\[
V_s = 356.3 \cdot h_g^{5/2} \cdot P_s
\]

\[
P_s = 0.1 \cdot h_g + P_0
\]

with \( h_g \) in meters (0 \( \cdot h_g \cdot 130)) , \( P_s \) in bars and \( V_s \) in m\(^3\) (0 \( \cdot V_s \cdot 4.6 \cdot 10^9\)).

Expressions [2] and [3] can be combined to suppress \( h_g \). The following expression of the gas storage volume can then be derived:

\[
V_s = 356.3 \left(10(P_s - P_0)\right)^{5/2} \cdot P_s
\]
The second difficulty results from the fact that the fluid is a compressible gas. The molar quantity stored, \(U_s\), depends on pressure, temperature and available volume. The gas is supposed to be perfect. So the perfect gas law can be applied:

\[
P_s \cdot V_s = U_s \cdot R \cdot T \tag{5}
\]

with \(P_s\) in Pascal, \(V_s\) in m\(^3\), \(U_s\) in moles, \(T\) in Kelvin degrees and \(R\) being the constant of perfect gas \((R = 8.314 \text{ J/mol. K})\). As the storage unit is not involved in the first configuration, let us derive the equation of the second configuration \((V_3 = 1, V_4 = 0)\):

\[
P_3 = P_4 \tag{6}
\]

\[
P_3 = P_s \tag{7}
\]

\[
\frac{dU_s}{dt} = d_p - d_c \tag{8}
\]

\[
P_s \cdot V_s = U_s \cdot R \cdot T \tag{9}
\]

\[
V_s = 353.6 \cdot P_s \left(10(P_s - P_0)^{5/2}\right) \tag{10}
\]

The flow rate \(d_p\) will be established through valve \(V_{R1}\), \((d_p - d_c)\) through valve \(V_3\), and \(d_c\) through \(V_7\) and \(V_{R2}\).

In the third configuration \((V_3 = 0, V_4 = 1)\), the equations \([6], [8], [9] \text{ and } [10]\) remain valid. Only the equation \([7]\) is modified and becomes:

\[
P_3 = P_s \tag{11}
\]

So the flow rate through valves \(V_{R1}\) and \(V_7\) is \(d_p\). It is 0 through \(V_3\), \((d_c - d_p)\) through \(V_4\), and \(d_c\) through \(V_{R2}\).

2.4. Input compressor equations

The input compressor has two different states: on and off. When the compressor is off \((V_1 = 0, V_2 = 1)\), the equations are:

\[
W_{cp} = 0 \tag{12}
\]

\[
P_2 = P_p \tag{13}
\]

On the other hand, when the compressor is on \((V_1 = 1, V_2 = 0)\), it is supposed to produce a constant specific work. Let us assume that it gives a compression rate
equal to 5. With the hypothesis of an ideal adiabatic compression weighted by $\eta = 0.8$, the work is given by:

$$W_{cp} = \frac{\gamma R T}{n(\gamma - 1)} \left( \frac{P_2}{P_p} \right)^{\frac{\gamma - 1}{\gamma}} - 1$$

[14]

with $\gamma = 1.31$ and

$$P_2 = 5P_p$$

[15]

### 2.5. Input dissipation valve equations

The valve is acting as a dissipator. Using the thermodynamic theory, a behavioural model is used under the assumption of an isenthalpic process. So, as the upstream and downstream pressures are known, the work is calculated by means of the following formula:

$$R T \ln \left( \frac{P_1}{P_2} \right) + W_{V_{ri}} = 0$$

[16]

Where $W_{V_{ri}}$ represents the internal variation in energy of the fluid.

### 2.6. Output compressor equations

The output compressor characteristics are identical to those of the input compressor (except for the compression rate which is equal to 2). Thus, when the compressor is off ($V_5 = 0, V_6 = 1$) the equations are:

$$W_{c_5} = 0$$

[17]

$$P_5 = P_4$$

[18]

And when the compressor is on ($V_5 = 1, V_6 = 0$), we have:

$$W_{c} = \frac{\gamma R T}{n (\gamma - 4)} \left( \frac{P_4}{P_5} \right)^{\frac{\gamma - 1}{\gamma}} - 1$$

[19]

with $\gamma = 1.31$.

$$P_5 = 2P_4$$

[20]
2.7. Output dissipation valve equations

The output dissipation valve is identical to the input one. The equation is:

\[ R \cdot T \cdot \ln \left( \frac{P_s}{P_5} \right) + W_{V_{R_2}} = 0 \]  \[21\]

Finally there are eleven unknown variables \(P_2, P_3, P_4, P_5, W_{V_{R_1}}, W_{V_{R_2}}, W_{C_p}, W_{C_c}, U_S, V_S, P_S\). However all the equations are not valid at the same time because the set of valid equations depends on the configuration. Let us consider the case where the storage is in the second configuration, and compressors and dissipation valves are on. Equations [6], [7], [8], [9], [10], [14], [15], [16], [19], [20] and [21] are valid. Thus there are eleven unknown variables and eleven independent equations. Table 1 summarises all the variables involved by the system.

| \(W_{C_p}\)   | Required energy by \(C_p\) |
| \(W_{C_c}\)   | Required energy by \(C_c\) |
| \(W_{V_{R_1}}\) | Dissipated energy from \(V_{R_1}\) |
| \(W_{V_{R_2}}\) | Dissipated energy from \(V_{R_2}\) |
| \(U_S\)       | Number of moles of gas in the storage |
| \(V_S\)       | Storage volume |
| \(P_S\)       | Storage pressure |
| \(T_S\)       | Temperature of the gas in the storage |
| \(R\)         | Constant of the perfect gas |
| \(P_0\)       | Reference pressure |
| \(P_P\)       | Production pressure |
| \(d_P\)       | Production flow rate |
| \(T_P\)       | Temperature of the gas in the production unit |
| \(P_C\)       | Customer pressure |
| \(d_C\)       | Customer flow rate |
| \(T_C\)       | Temperature of the gas in the customer unit |
| \(P_2\)       | Pressure after \(C_p\) |
| \(P_3\)       | Pressure after \(V_{R_1}\) |
| \(P_4\)       | Pressure before \(C_c\) |
| \(P_5\)       | Pressure before \(V_{R_2}\) |
| \(V_1...V_6\) | On/off valves |
| \(T\)         | Temperature of the gas through the unit |

**Table 1. Summary of the variables**

2.8. Control policy of the production unit
Initially, the pressure in the storage ($P_s$) is equal to 56 bars, the storage is in the first configuration, and the input and output compressors and dissipative valves are off.

The control law of the storage unit has to satisfy a set of constraints. The first one is about the input compressor: if $P_p < P_3$ then the compressor $C_p$ has to be on, otherwise the gas will flow back from the storage unit to the production unit. The second constraint involves the output compressor: if $P_4 < P_c$ then the compressor $C_C$ is on, otherwise gas will flow back from the consumer to the storage unit.

Another set of constraints involves the storage unit configurations. If $d_p = d_c$ then the storage has to switch to the first configuration. If $d_p > d_c$ then the storage unit has to switch to the second configuration and, according to the values of $P_p$ and $P_S$, the input compressor will be activated or not. If $d_p < d_c$ then the unit has to switch to the third configuration and, according to the values of $P_p$ and $P_S$, the output compressor will be activated or not. Satisfying these constraints means that a supervisory control function has to test these variables and to open or close the on/off valves accordingly. Some security constraints also restrict the pressure $P_S$ to vary between a minimal and a maximal value. When the maximal value is reached, the storage S has to be isolated from the production (fourth configuration).

For further details about the these constraints and some hints about production and consumption values, the reader can refer to [CHA 98a]. An example of typical evolution of the storage pressure $P_S$ in function of the time is represented in [VAL 98, figure 6].

3. Hybrid modelling

3.1. The issue

A discrete event model of a system is a model in which the variables and the time take their values in a finite set or in $\mathbb{N}$ (set of natural numbers). A continuous model of a system is a model in which the variables and the time take their values in $\mathbb{R}$ (set of real numbers). Finite automata and Petri nets are examples of discrete event models whereas differential algebraic equations are the basic elements of continuous models. The challenge for modelling hybrid systems is to combine the two kinds of models.

For instance, in the gas storage example, $P_S$, $V_S$, $U_S$ are continuous variables whose values have to be known at each time point whereas $V_1$, $V_2$ etc. are Boolean variables (open or closed) whose values only change at specific time points (the events) when the configuration of the system is modified. How can this be done?

Without presenting the whole model of the gas storage system, we will use some significant fragments of it in order to depict how it can be elaborated by means of the three principle approaches: Coloured Petri nets, Hybrid Petri nets and Petri nets with Differential Algebraic equations. In particular, we will omit the behaviours of the compressors and of the dissipators. In this section, we will only study the representation of the stored quantity of moles $U_S$, assuming that the configuration changes are independent of $P_S$ (except for the discussion about Coloured Petri nets).
In section 4, this characteristic of the gas storage unit will be taken into account to better discuss the advantages and limitations of these approaches.

3.2. Modelling principles with Coloured Petri nets

In a way, it could be considered that it is not a problem to introduce continuous variables in a Petri net. As a matter of fact, complementing Petri nets with timing considerations (timed or time Petri nets) and attaching data (possibly continuous) to tokens as attributes (high-level Petri nets such as Coloured Petri nets or Predicate-Transition nets) are well known ideas which are currently been applied in the specific context of hybrid system modelling [GEN 98, HAN 96, LEM 98]. A high-level Petri net with a complex data structure attached to the tokens and durations attached to the transitions can indeed capture a large range of hybrid phenomena. However, it must be pointed out that, even if time takes its values on the set of real numbers \( R \), the values of the variables attached to the tokens are only modified when the tokens are involved in some transition firings i.e. at discrete time points. This is why, typically, in such models some transitions are fired for each \( \Delta t \) or for each small variation of a continuous variable in order to regularly update the token attributes.

If we consider the gas storage example, the evolution of the number of moles \( U_s \) can be modelled by the coloured Petri net in figure 7. The token in place \( d_p \) (respectively \( d_c \) and \( U_s \)) contains the corresponding variable \( d_p \) (respectively \( d_c \) and \( U_s \)). The transition \( t_p \) (respectively \( t_c \)) is fired each time the production flow rate \( d_p \) (respectively consumer flow rate \( d_c \)) is modified. Transition \( t_u \) is fired each time \( t \) is increased by \( \Delta t \). From the values of \( d_p \), \( d_c \) and \( U_s \) at time \( t \) and equation [8], the value of \( U_s \), at \( t + \Delta t \), is calculated as follows:

\[
U_s(t + \Delta t) = U_s(t) + \left(d_p(t) - d_c(t)\right) \Delta t
\]

[22]
Then from equations [9] and [10], $P_S$ and $V_S$ can be computed. If $\Delta t$ is sufficiently small then the approximation of $U_S$, $P_S$ and $V_S$ will be accurate. Transition $t_f$ is fired when the abnormal thresholds of $P_S$ are reached.

This approach has several drawbacks. The major one is that if transitions $t_p$ and $t_c$ correspond to actual events of the system, transition $t_u$ is artificial because it is only used to calculate $U_S$. In the case of a sampled system it is natural to have a transition which is fired each $\Delta t$ [DEM 96]. If this is not the case, $t_u$ might be confusing. Another drawback is that integrating a system of equations with a fixed and small step is not efficient, modern methods have variable integration steps. It must be pointed out that the value of $P_S$ has to be known at each time point in order to fire $t_f$ when the thresholds are reached. In consequence, it is not possible to only compute $U_S$ (by means of an algebraic expression) at the time points corresponding to the firings of $t_p$ and $t_c$. It is necessary to compute it at each $\Delta t$.

In front of these difficulties two classes of approaches have been developed: hybrid Petri nets and Petri nets with differential algebraic equations. In contrast to the case of high-level nets, in these two approaches, which have been studied in the MENESR project, the continuous variables are known at each time points (i.e. continuously). In the first case, the continuous variables are represented by the continuous markings of the continuous places [ALL 98b, BAI 91, DAV 87, FLA 96], whereas, in the second, a system of differential algebraic equations is integrated between two transition firings [AND 94, CHA 98b, DAU 94, VAL 93, VIB 97, VAL 98]. Either the continuous variables are assumed to be global ones, or they are assumed to be token attributes in a Predicate Transition net, but, these attributes are supposed to continuously vary along the time because they are the result of the integration of the equations.

3.3. Modelling with Hybrid Petri nets

In order to introduce this approach and the following one, we assume that the thresholds of $P_S$ are never reached and that the gas storage is never isolated because its pressure is too high or too low. This difficulty will be considered in section 5.
Figure 8. Hybrid Petri net

The left part of figure 8 (see [ALL 98b]) describes the discrete part, i.e. the configuration changes. The continuous place P1 represents Us, the input flow in configuration 2 is the flow attached to transition T1 (dp - dc), and the output flow in configurations 3 or 4 is the flow attached to transition T2 (dc - dp). As the flows dp and dc are assumed constant on pre-defined time intervals, it is clear that the marking of place P1 can be computed at any time point by linear interpolation. As a matter of fact, equation [8] is directly implemented by the sub-net (T1, P1, T2).

The advantages compared to a modelling based on coloured Petri nets are the following:

- there is a clear representation of the configurations and of the physical flow of gas,
- no primitives in the model are uniquely motivated by the representation of how the computation is done (such as transition t0 in figure 7),
- the continuous variable Us is known at any time without the necessity of computing it at each Δt.

As a matter of fact, a Hybrid Petri net can be systematically transformed into a specific class of hybrid automata [ALL 97, ALL 98a] which is not the case of Coloured Petri nets in which there is no explicit differential equation. The Coloured Petri net can be transformed into automata, possibly with an infinite number of states but their states can be enumerated which is not the case of hybrid automata.

3.4. Modelling with Petri nets and differential algebraic equations

The two approaches [CHA 98b] and [VAL 98] share a common basic principle: continuous variable evolutions are directly and globally defined by a set of differential algebraic equations (DAE). This set of equations is defined from the Petri net marking, and the transition firing is triggered by some threshold of the continuous variables. Basically, the principle of hybrid automata [ALU 95] has been adopted in the context of Petri nets. The differences are that in [VAL 98] (DAE supervised by Petri nets) some equations may directly be attached to markings, whereas in [CHA 98b] (Predicate transition nets with DAE) they are attached to places because the continuous variables are not global but locally attached to the tokens. In [VAL 98] the main objective is simplicity, generality and compactness of the model, whereas modularity is the major concern in [CHA 98b] in order to eventually cope with complex systems having a large number of reachable markings. In the simple case of Us computation, the two approaches would be the same.
A possible model is represented in Figure 9. The four places describe the four possible configurations of the gas storage. An equation is attached to each place in order to compute $U_s$ as follows:

$$\frac{dU_s}{dt} = 0 \text{ for } P_1, \quad \frac{dU_s}{dt} = d_p - d_c \text{ for } P_2, \quad \frac{dU_s}{dt} = d_p - d_c \text{ for } P_3 \text{ and } \frac{dU_s}{dt} = -d_c \text{ for } P_4.$$ 

After each transition firing, a new set of equations is built and a solver is called in order to compute the variations of the continuous variables. When a pre-defined event or a threshold of a continuous variable is reached, the solver is stopped and the corresponding transition is fired. In consequence, even if the continuous variables are assumed to be token attributes, their values are known at each time point and not only at each transition firing.

The major advantage of this approach is its generality: any system of differential algebraic equations which can be solved by a powerful solver (Gear method for example) can be used. The complexity of the continuous part of the hybrid model is completely separated from that of the discrete one. Furthermore, the approach is not limited to pre-defined classes of differential equations as it is the case for hybrid Petri nets [BAI 91, DAV 87, FLA 96] or batch Petri nets [PRU 96, CAR 98]. The major difficulty is that if you gather a set of differential algebraic equations (attached to different places which happen to simultaneously contain a token) without any precaution, it will probably not have any solution. In particular, the initial values of the variables and of their derivates have to be consistent with the algebraic equations.

4. Difficulties pointed out by the benchmark

The aim of the benchmark is not only to point out the advantages of each approach, it is also to determine their limits. The choice of the gas storage example is motivated by the simplicity of its discrete part. Although it remains relatively simple from the point of view of the continuous equations (simple solvers are sufficient for
the computation of the continuous variables), it is sufficiently complex to illustrate some difficulties. The complete model of the gas storage by means of hybrid Petri nets [ALL 98b], differential algebraic equations supervised by Petri nets [VAL 98] and predicate transition nets with differential algebraic equations [CHA 98b] can be found in the corresponding papers and will not be completely detailed here. Two difficult points will be illustrated in the sequel. The first one concerns the difficulty of dealing with non linear algebraic equations without using (explicitly or implicitly) a solver. The second is the difficulty of designing the model according to a modular approach, when no decomposition of the discrete part into subnets (fragments of a global Petri net) can guarantee that each continuous variable is only involved in one sub-Petri-net (presence of continuous variables shared between the different discrete modules).

4.1. Non linear algebraic equations

Ordinary Petri nets are well suited to represent sequences of events and activities (discrete event systems), but they are inadequate to describe Boolean expressions. Indeed, the "logical and" of two sensor values in a receptivity attached to a transition in a Grafcet or an interpreted Petri net is of a different nature than the "Petri-net and" implemented by two places which are the input of some transition. The sensor values are not modified when the receptivity is tested or when the corresponding transition is fired whereas the tokens of the input places are removed at the the firing instants. The underlying logic expressed by Petri nets is not classical logic, it is linear logic [GIR 97, VAL 97].

In the same way, it seems that continuous places and transitions are well suited to represent differential equations (especially if they are linear), but they are inadequate for describing algebraic constraints. For example in equation [8]:

$$\frac{dU_i}{dt} = d_P - d_C$$

it can be said that $d_P$ produces some $U_S$, and $d_C$ consumes some stuff from $i$; in the same way the input transition of a place produces tokens in it and its output transition consumes tokens. This is the seminal intuition behind Hybrid (and Continuous) Petri nets. In contrast, when equations [4] and [5] are used to compute $P_S$ from $U_S$, nothing is produced or consumed; the variables are read and written and this cannot be simulated by Petri net like dynamics.

Not surprisingly, the algebraic constraints [4] and [5] make it difficult to model the gas system by means of a Hybrid Petri-net. These equations have to be used outside the model in order to derive $P_S$ from $U_S$ and check if $P_S$ does not reach the specified thresholds. Equations [4] and [5] could be considered as marginal because $P_S$ is not used to compute $U_S$. However, as the algebraic constraints are non linear, an accurate monitoring of the thresholds requires $U_S$ and $P_S$ to be computed at any time points. No linear interpolation is possible. One of the advantages of Hybrid Petri nets over Coloured Petri nets is lost because it is also necessary to compute the continuous variables at each $\Delta t$, with $\Delta t$ as small as possible to obtain a good precision. This computation remains implicit and is not represented in the hybrid Petri net model.

In the gas storage example, there is only one differential equation. It is clear that if there are several variables which are derived from differential equations and if the
algebraic constraints simultaneously involve these variables, the difficulty will increase and solvers for algebraic equations will be necessary.

No such problem arises when Petri nets with differential algebraic equations are used [CHA 98b, VAL 98]. Indeed, a solver based on a method such as the Gear approach, simultaneously addresses differential and non linear algebraic equations. As it has been mentioned above, the complexity of the continuous part and that of the discrete one are split as most as possible. A solver is called to compute the date of the next transition firing (threshold of a continuous variable or its derivative) from a global set of equations. The choice of the solver is totally separated from the modelling issue, and only depends on the complexity of the continuous part. It is used to provide the accurate value of the continuous variables at transition firing. The discrete part evolution is achieved by a Petri net "player".

4.2. Possibility of hybrid interactions between the equipments

Another complex issue is the elaboration of a modular approach. In simple cases, the interactions between the equipments are mainly the consequence of the discrete view. Indeed, interactions are based on the fact that a configuration change of an equipment systematically leads to a configuration change of another one. For example, at the end of a transfer operation between two reactors, one of the reactor switches from the configuration "being emptied" to "isolated" and simultaneously the other one passes from "being filled" to "isolated". Once more, the gas storage example is relatively simple because the storage unit is the only equipment with a non trivial complexity (4 configurations plus a failure state). The other devices (the two compressors and the two dissipation valves) have only two control states (on or off) and even if equations [14], [16], [19] and [21] are non linear, they are only used to compute the energy required for optimisation and for energetic balance.

In order to interconnect the model of each equipment and to derive the model of the whole system, the approach developed in [AND 96] is based on transition merging. It is therefore assumed that the interactions between the various equipments occur only when their states change. For instance, if the production policy changes (for example \( d_p \) was equal to \( d_c \) and becomes greater than it), it is possible that the storage unit passes from configuration 1 to configuration 2 and that the compressor \( C_p \) has to be started simultaneously because the production pressure \( P_p \) is less that the storage pressure \( P_s \). In figure 10, a fragment of the \( C_p \) model and of that of the storage unit are represented. If the above behaviour was systematic, it would be possible to merge the transition denoting the state change "off-on" of \( C_p \) (T2) with the one denoting storage unit state change "from config 1 to config 2" (represented by T12). However, this is not the case because various situations are possible when a state change occurs. The equipment state changes are linked to the value of \( d_p \), and also to those of \( P_p \) and \( P_s \), while \( P_p \) depends on the production characteristics and \( P_s \) depends on parameter \( P_0 \) in equation [4]. Anyway, the model of the discrete view should be the same whatever the continuous parameters of physical devices (such as \( P_0 \)) are.

For the bottom-up approach adopted in [CHA 98b] in which the continuous variables are distributed on the tokens (the state variables of each equipment are associated with the tokens of the nets describing their configurations), this kind of configuration changes and interactions between the equipments (when transition
merging is not possible) is a serious problem. Actually, variable \( P_5 \) is associated to the token contained in \( P_{11} \) which is not an input place of transition \( T_2 \) (hybrid interaction). If this transition cannot be merged with \( T_{12} \), \( P_5 \) should also be associated to the token in place \( P_2 \). In other words, the models of the compressor \( C_P \) and the storage unit have to share a common variable \( P_s \). With a transition merging approach, the merged transition would be connected to two input places, therefore, it can have access to both the variables of the storage unit and those of the compressor \( C_P \). The fact that the models of the equipment share common variables will make it difficult to guarantee that the global equation system resulting from the dynamic association of sub-systems is homogeneous (because it will not be the direct consequence that the sub systems are homogeneous for each equipment).

This is the reason why the model of the gas storage system in [CHA 98b] seems to be made up of 5 independent Petri nets which are, in fact, state machines (the storage unit, the two compressors and the two dissipation valves). Actually, these Petri nets are interconnected by means of shared continuous variables, and this interconnection is not expressed by the discrete view although the transitions involved in the models of the discrete parts of various equipments may, sometimes, simultaneously fire.

This issue is not a problem for the approach presented in [VAL 98]. The main reason is that all the variables are assumed to be global and can be used in the conditions attached to all the transitions. The Petri net which contains the places denoting the four configurations of the storage unit also acts as a supervisor of the whole storage system. Some places and transitions only represent the sequences of tests which are necessary to determine the configuration of all the equipments of the system. The places corresponding to the steps of the decision process have no attached equations and the the tokens leave the places as soon as they arrive. This is the case of places \( P_1 \) and \( P_2 \). in the fragment of the model shown in figure 11; only places \( P_6 \) and \( P_7 \) denote system configurations.

**Figure 10. Fragment of predicate transition net with differential algebraic equations**
P1 is the place which contains one token when a decision process starts and P2 models one of the four possible flowrate configurations (dp=dc, dp<dc, dp>dc, dp=0). Transitions T5 and T6 represent the decision concerning the pressure conditions. P6 denotes configuration 1 and P7 represents the abnormal configuration when compressors cannot compensate the lack of production pressure.

![Diagram](image)

**Figure 11. Fragment of DAE supervised by PN**

The complete model [VAL 98] corresponding to this approach has three Petri nets running in parallel, each modelling one of the physical phenomena (valves configurations, input compression, output compression). The interaction between these three Petri nets is implemented by means of the continuous variables which are considered as global variables. At each instant, only one place of each Petri net is marked to indicate the state of the corresponding phenomenon.

As long as there is no attempt to develop a bottom-up approach (a hybrid Petri net for each equipment which are interconnected to derive the global model), there will also be no specific problem for the hybrid Petri net based method.

5. Conclusion

This paper has presented a benchmark example which can be used to compare various approaches for hybrid modelling. It is a gas storage system, which has been simplified from the point of view of the discrete aspect. The aim is to point out the differences between these approaches when the continuous part is not trivial *i.e.* when it includes non linear algebraic constraints and when the interactions between the equipments depend on some continuous variables as well as on some event occurrences.

As an elementary guideline in order to classify the approaches, we can give the following hints:
- Hybrid Petri nets provides a graphical representation of the continuous flow of material, whereas in Petri nets with differential algebraic equations, the graphical form is only used to represent the discrete part.
- Hybrid Petri nets are not very well suited when the continuous part contains non linear algebraic constraints, whereas Petri nets with differential algebraic equations establish no restriction on the way the continuous part is represented. Any efficient solver can be used.
- For Petri nets with differential algebraic equations, the choice of the solver is completely disconnected from the modelling issue; this is not the case for coloured Petri nets. As the form of the differential equations attached to the continuous places of a hybrid Petri net is pre-defined, generic simulation tools can easily be built without requiring any other general purpose solver. However, if non linear algebraic constraints are added, this advantage is lost.
- Predicate transition nets with differential algebraic equations allow a bottom-up approach, but this approach turns to be more difficult when the equipment models include shared variables. In this case, it will be difficult to be sure that non solvable systems will never be obtained in some situations. This problem of obtaining non solvable differential algebraic systems does not exist with hybrid Petri nets (without any additional algebraic constraints) because the restrictions imposed by the model guarantee that a simple solvable system is always obtained.
- Differential algebraic equations supervised by Petri nets is the approach which is the most robust to non trivial continuous part. Its major limitation would be met with systems having a discrete part such that the number of reachable markings is very large. Then a structured approach for which the equations and the variables are attached to sub-nets would probably be necessary.

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7. References


